GENETIC ALGORITHM FOR ION THRUSTER GRID DESIGN

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Abstract

A genetic algorithm was used to optimize ion thruster grid sets with regard to maximizing impulse per unit area, essentially equivalent to maximizing propellant throughput capability per unit area. The genetic algorithm presented herein made use of the ffx ion optics simulation code for grid lifetime predictions. Grid sets were optimized for several combinations of net accelerating voltage and current density, and grid feature recommendations are made concerning how future missions can be met.

I. ION OPTICS

One component of the ion thruster that may be life limiting is the ion optics assembly, or grids, used to accelerate ions from the discharge chamber to produce thrust. Present ion optics designs for deep space missions consist of two grids as shown in Figure 1.

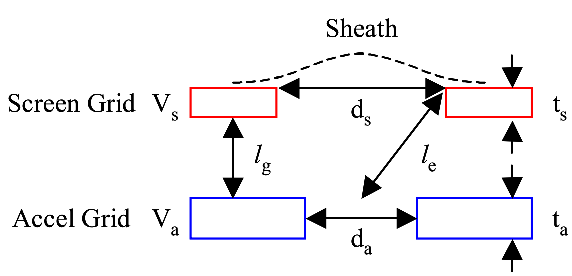

Figure 1. Two grid ion optics setup.

Perveance, P {A/V[1.5]}, is a measure of the current density, j_p, passing between two surfaces according to a specified potential difference and separation. The perveance fraction, as applied to ion optics, is given in Eq. (1). A perveance fraction of unity indicates that the maximum possible current density is being extracted for the given total accelerating voltage, V_T, and effective ion acceleration length, l_e. The downstream current density, j, is lower than the upstream current density, j_p, by the factor of the ion transparency, ϕ.

$$f_p = \frac{j_p}{P_{Max}} \frac{l_e^2}{V_T^{3/2}} \qquad (1)$$

$$P_{Max} = \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{m_i}} \quad \left\{\frac{A}{V^{3/2}}\right\}$$

$$l_e = \sqrt{\left(l_g + t_s\right)^2 + \tfrac{1}{4} d_s^{\,2}} \quad , V_T = V_N - V_a$$

In this work, the ffx ion optics code is used to provide grid lifetime predictions. Charge exchange ions erode both the accel grid hole barrel, increasing the likelihood of electron backstreaming, and the downstream side of the accel grid, often resulting in a pit and groove erosion pattern. To encompass several possible failure mechanisms, the end of life of the accel grid is taken here to be the time at which 50 percent of the accel grid mass has been worn away.

II. EVOLUTIONARY ALGORITHM

Evolutionary algorithms mimic biological evolution to seek problem solutions. Genetic algorithms and evolution strategies are two similar sub categories of evolutionary algorithms. The general overview of an evolutionary algorithm is presented in Figure 2. The algorithm works with a population of solutions called chromosomes. Each chromosome codes for a set of values for the unknown variables of the problem.

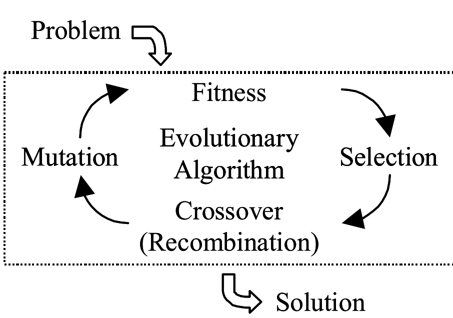

Figure 2. Outline of an evolutionary algorithm.

Initially, on generation zero, the chromosomes are generated randomly. The ffx code is used to evaluate the chromosomes and assign them fitness values. Selection, crossover, and mutation are then used to form the next, hopefully improved, generation. Genetic algorithms tend to use binary value encoding, large chromosome populations, and favor crossover over mutation. Evolution

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strategies tend to use real value encoding, smaller populations, and favor mutation over recombination.

The evolutionary algorithm was used here for optimization, maximizing impulse per unit area, given in Eq. (2). This is the thrust provided by an aperture multiplied by the operation time, normalized to its area. As input to the problem, the double to single current ratio was set to zero and xenon was chosen as the propellant. Furthermore, this quantity was maximized for combinations of current density of 25 and 50 A/m^2 and net accelerating voltages of 1000, 1800, and 3000 V. After these reductions, the fitness value, Eq. (3), was chosen as the current density (an input) multiplied by the predicted operation lifetime multiplied by the thrust factor.

$$\frac{F \cdot L}{A} = \frac{J_b}{A_g} L \frac{m_i}{e} \frac{1+\frac{\sqrt{2}}{2}\frac{J_b^{++}}{J_b^{+}}}{1+\frac{J_b^{++}}{J_b^{+}}} ft \sqrt{\frac{2eV_N}{m_i}} \tag{2}$$

$$J_b^{++}/J_b^{+} = 0 \qquad xenon \rightarrow m_i$$

$$j = 25, 50\ A/m^2 \qquad V_N = 1000, 1800, 3000\ V$$

$$f = j \cdot L \cdot ft \tag{3}$$

The ffx code lifetime predictions were calculated from beginning of life erosion rates. To guarantee electron backstreaming was mitigated over the entire lifetime, a 40 V margin against electron backstreaming was required of acceptable solutions. This was seen to be sufficient following full lifetime simulations of the algorithm solutions. An additional input to the problem was a specified (typical) physical screen grid open area fraction, ϕ_s, of 66.7 percent. Acceptable algorithm solutions were required to have ion transparencies greater than or equal to this physical transparency.

Table 1 reports the methods used in the evolutionary algorithm applied to this problem [1-2]. The methods of intermediate recombination and mutation for real value chromosome encoding were described by Hartmut [3]. The adaptive crossover and mutation schemes used here were similar to those described by Srinivas [4].

Table 1. Guidelines for the evolutionary algorithm.

Population	25
Encoding	Real Value
Scaling	Rank Selection. Selective Pressure = 2.0
Selection	Roulette Wheel
Elitism	Yes, Best Chromosome
Crossover	Intermediate Recombination. Rate = 50 to 75 %
Mutation	$R \approx 0.5$, $k = 8$. Rate = 1.5/6

Each chromosome coded directly for six variables:

- screen grid thickness (t_s)
- screen grid hole diameter (d_s)
- grid spacing (l_g)
- accel grid thickness (t_a)
- accel grid hole diameter (d_a)
- accel grid voltage (V_a).

Indirectly, three values were set at run time according to the individual chromosome variables:

- aperture center to center spacing (l_{cc}), through the physical screen grid transparency (ϕ_s)
- beamlet current (J_b), through the specified current density (j) and center to center spacing (l_{cc})
- perveance fraction (f_p), using t_s, d_s, l_g, and V_a.

Table 2 lists the minimum and maximum variable values for a given evolutionary algorithm. The perveance fraction equation was used to determine upper limits for the geometrical variables (t_s through d_a) using Eq. (4), while the lower limits were set somewhat arbitrarily. For the minimum grid spacing, a maximum electric field of 3 kV/mm was imposed. Minimum values based on $l_{g\ min}$ might instead be chosen. With regard to accel grid voltage, the R ratio was allowed to vary between 0.85 and 0.90.

$$l_{e\max} = \sqrt{\frac{P_{Max}\left(V_N/0.85\right)^{3/2}}{j/\phi_s}} \tag{4}$$

Table 2. Allowed variable ranges.

Variable	Lower Limit	Upper Limit
t_s	$\frac{1}{10} l_{e\max}$	$\frac{1}{2} l_{e\max}$
d_s	$\frac{1}{10} l_{e\max}$	$2 l_{e\max}$
l_g	$\dfrac{\frac{V_N}{R_{\min}} - V_d}{E_{\max}}$	$l_{e\max}$
t_a	$\frac{1}{10} l_{e\max}$	$\frac{1}{2} l_{e\max}$
d_a	$\frac{1}{10} l_{e\max}$	$2 l_{e\max}$
V_a	$V_N\left(1-\frac{1}{R_{\min}}\right)$	$V_N\left(1-\frac{1}{R_{\max}}\right)$

III. RESULTS

A typical progression of the algorithm is shown in Figure 3 for a net voltage of 1800 V and a current density of 50 A/m^2. In each generation there are generally two groups. The chromosomes in one group are similar to the best chromosome and are searching for better solutions in a local region using less crossover and mutation. The chromosomes in the other group are subject to heavy crossover and mutation and are searching broadly to avoid a local maximum solution.

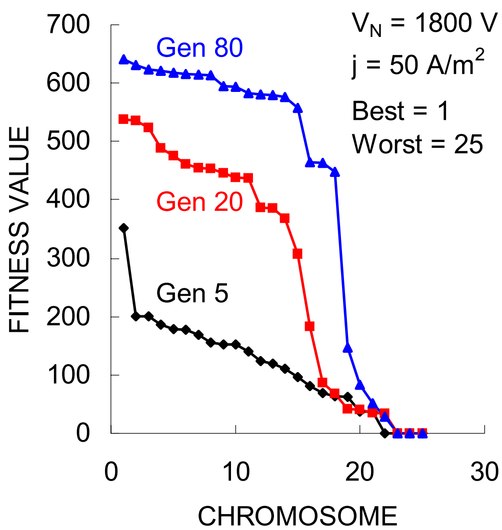


Figure 3. Chromosome fitness value distribution as a function of generation.

Figure 4 shows the variable values of the best chromosome in the population on every generation. The algorithms were typically run for 60 to 150 generations until convergence was observed.

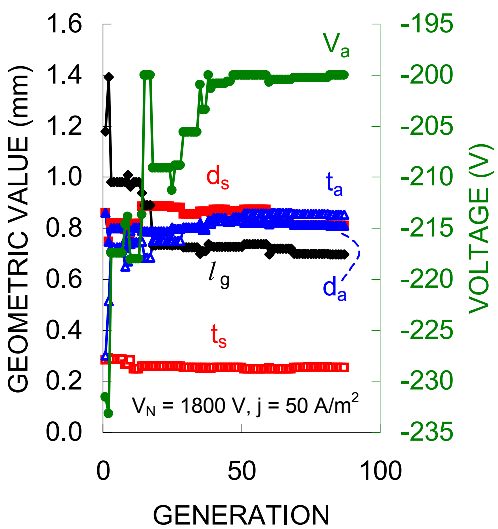


Figure 4. History of the most fit chromosome in each generation.

The best six chromosomes from generations 0 and 65 are shown in Figure 5. For any net voltage (V_N) and current density (j) combination, the algorithm always found the same basic solution exemplified in this figure. This solution has the following properties:

- Least negative V_a
- Minimized l_g
- Minimized t_s
- Remaining three variables (d_s, d_a, t_a) between limits
    - d_a was required to be smaller than or equal to d_s. The typical solution tended to maximize d_a under this constraint, i.e. $d_a = d_s$.

This solution was found for the following reasons:

- V_a: Minimize the sputter rate by reducing ion impingement energy.
- Small t_s, l_g, and d_s: Went to a low perveance fraction, f_p. Going to a low perveance fraction has the effect of funneling a larger fraction of charge exchange ions created in the intra grid region through the accel grid hole rather than into the accel grid. Also, a small t_s helps to maintain a high ion transparency.
- d_a: Went to a large d_a to reduce the intra grid neutral density, which lowered charge exchange ion production. This result is not immediately intuitive because alternatively going to a small d_a would increase the amount of accel grid material available to be sputtered.
- t_a: Went to a relatively large, but not maximized, accel grid thickness. This increases the amount of accel grid mass available for removal, extending predicted lifetime.

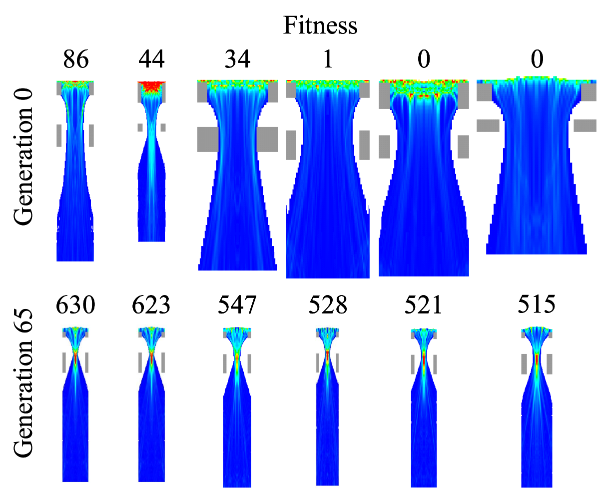


Figure 5. The best six chromosomes from generation 0 and 65.

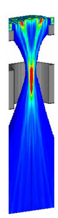
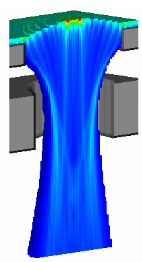
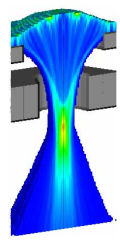
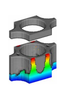
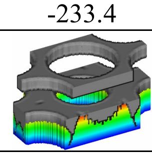
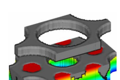
There are three possible drawbacks to a low perveance fraction solution. First, the holes are small, increasing the likelihood of hole misalignment. Second, the solution operates near the crossover limit, which constrains the amount by which the current density can be reduced. This can be troublesome in thrusters with low flatness parameters. Third, operating at a low f_p indicates a high divergence angle (α) or equivalently a low thrust factor (ft). Thrust factor had a minor affect on fitness compared to predicted lifetime (L).

One fix for these concerns is to search for a solution that has a higher perveance fraction, which will operate further away from the crossover limit.

Limiting acceptable solutions to those with perveance fractions greater than or equal to 0.5 resulted in solutions with increased grid spacing (l_g) values. While these solutions operated away from the crossover limit, misalignment was still likely as they still had small screen grid hole diameters (d_s).

To fix the small aperture problem, the grid spacing (l_g) was fixed at its minimum value. The resulting common solution is shown as "Medium f_p" in Table 3. This solution has large holes, making alignment easier, and operates at a relatively high perveance fraction, which indicates it can be throttled to lower current densities.

Table 3. Evolutionary algorithm solutions.

			
Name	Low f_p (typical)	Medium f_p	NEXT
f_p (-)	0.19	0.50	0.29
F ($A \cdot yr/m^2$)	630	101	76
l_g (mm)	0.698 (min)	0.698 (fixed)	
V_a (V)	-200 (max)	-233.4	-210
50 Percent Accel Grid Mass Loss			
F ($A \cdot yr/m^2$)	273	135	104

The "Medium f_p" solution was fundamentally different than the "Low f_p" solution. First, the accel grid voltage had to be slightly more negative to resist electron backstreaming. Second, the screen grid thickness (t_s) was not minimized. Third, the accel grid thickness (t_a) was maximized. Finally, the accel grid hole diameter (d_a) was slightly reduced in size relative to the screen grid hole diameter (d_s), whereas in the "Low f_p" solution d_a was always basically equal to d_s.

Table 4 reports the average grid geometries found using the evolutionary algorithm normalized to the screen grid hole diameter (d_s). Compared to the NEXT grid geometry, the algorithm solutions generally had larger accel grid hole diameters (d_a) and thicker screen (t_s) and accel (t_a) grids. The grid spacing value reported for the NEXT grid is halfway between the cold and hot (operating) grid gaps.

Table 4. Average grid geometry ratios for all evolutionary algorithm solutions.

	Low f_p	Medium f_p	NEXT
l_{cc}/d_s	1.16	1.16	1.16
t_s/d_s	0.27	0.24	0.20
d_s/d_s	identity	identity	identity
l_g/d_s	0.68	0.27	0.25
t_a/d_s	1.01	0.50	0.40
d_a/d_s	0.97	0.78	0.60
f_p	0.18	0.51	0.29

IV. SUMMARY

The ffx code was used in conjunction with an evolutionary algorithm to design several grid sets for the goal of maximizing grid lifetime. Additional information could be used with these results to select an appropriate grid set geometry for an ion thruster. For instance, selecting a thruster diameter and subsequently determining the discharge chamber flatness parameter would indicate what range of current densities to expect over the grid face. Additionally, the desired spacecraft velocity change gives an indication for the ion accelerating voltage. Taken together, such information could allow the centerline hole (where the current density might be greatest) to be sized using the "Medium f_p" solution for example.

V. REFERENCES

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